Cerf Replies: In the Letter [1], I consider a Monte Carlo technique for the calculation of thermodynamic properties of nuclear systems with a monopole pairing interaction. The method is based on a stochastic sampling of paths in a many-body configuration space, which, using an interaction picturelike decomposition of the many-body propagator, makes it possible to eliminate the time-discretization error inherent in most path-integral Monte Carlo methods at finite temperature. The *direct* sampling of many-body states provides a natural approach to the computation of thermodynamic variables in the canonical ensemble. For example, the energy and the specific heat are calculated from the measured moments of the number of hops in the sampled configuration paths.

In their Comment [2], Rombouts and Heyde rightly point out the importance of nonaccompanied particles in the description of finite-temperature properties, particularly in the case of a $(h_{11/2})^6$ configuration. I take this opportunity to clarify that the case treated in [1] is intended as an illustration of the method only. In this example, nonaccompanied particles are purposely *not* included; the procedure, therefore, gives the exact result but in a *smaller* model space. However, as implied by [1], the Monte Carlo method based on configuration paths is not restricted to this particular example. It can be used as well for treating the full N-particle space of a $(h_{11/2})^6$ configuration that is considered in [2], as I shall discuss in this Reply.

In order to demonstrate the use of the Monte Carlo procedure, I consider in [1] a half-filled shell ($j = \frac{11}{2}$) with a constant pairing strength G within the quasispin formalism. Accordingly, I consider a model in which all particles are accompanied in the simulation presented, so that the calculation involving the fully accompanied states performed in [2] (cf. the dashed lines in Figs. 1 and 2) exactly reproduces my results (see Figs. 1 and 2 in [1]). However, the technique discussed in [1] can be applied to the sampling of many-body states with nonaccompanied particles equally well, as emphasized in the discussion following Eqs. (1) and (4) in [1]. The sampling of the full N-particle space then requires using paths involving paired nucleons (behaving as bosons with an attractive interaction *and* unpaired nucleons (acting like classical particles, as they do not hop under the action of the considered pairing Hamiltonian). The treatment of unpaired particles in the Monte Carlo simulation amounts to adding a constant contribution to the action $A[C(t)]$, which is equal to the inverse temperature β multiplied by the (constant) energy of the unpaired particles in the path $C(t)$. Of course, the nucleon pairs can hop only to the pairs of conjugate orbits that are not occupied by unpaired nucleons, so that the sampling of pair-configuration paths must take this blocking effect into account.

Thus, if a larger model space is considered which includes the states with nonaccompanied particles as in [2], the sampling of paths $C(t)$ can be performed straightforwardly by using $E^{1b}(C) = -GN_p(C)/2$ rather than $-GN/2$, where $N_p(C)$ is the number of paired nucleons in configuration C, so that the one-body energy E^{1b} is no longer independent of C. (Here, G is constant and we have $\epsilon_k = 0$.) Accordingly, it is necessary to use an additional weight $\exp(-\beta E^{1b})$ in the sampling, while this weight was constant—and, therefore, ignored—in the example shown in [1]. The Monte Carlo simulation is then based on the sampling of periodic sequences of C's of variable length D (with $D \neq 1$) *and* variable number of paired nucleons N_p, with probability

$$p \propto \exp(\beta G N_p/2)\, G^D \beta^D / D! \tag{1}$$

Also, the Monte Carlo estimate of the energy must be written as

$$\langle H \rangle_\beta \simeq -G\langle\langle N_p \rangle\rangle/2 - \beta^{-1}\langle\langle D \rangle\rangle, \tag{2}$$

where $\langle\langle D \rangle\rangle$ and $\langle\langle N_p \rangle\rangle$ stand for the simulation average of the number of hops and paired particles over the sampled paths, respectively. The first term on the right-hand side of Eq. (2) was constant in the example treated in [1], as then $N_p \equiv N$. I have checked that the results of a Monte Carlo simulation based on such a sampling of configuration paths including $N_p < N$ (with $N = 6$, $G = 1$ MeV, and $\Omega = j + \frac{1}{2} = 6$) coincide with those of [2] in the full space, as expected. A simple adaptation of the Monte Carlo program used in [1] is necessary in order to account for nonaccompanied particles, and the computation is conceptually identical.

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